Section I Multiple Choice (10 Marks)

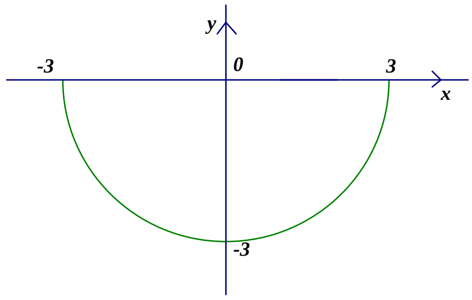
Attempt Questions 1 – 10 (1 mark each)
Allow approximately 20 minutes for this section.

Question 1

The condition for the quadratic equation $3x^2 - 12x + k = 0$ to have real roots is

A) $k \leq 36$ B) $k \geq 36$ C) $k \leq 12$ D) $k \geq 12$

Question 2

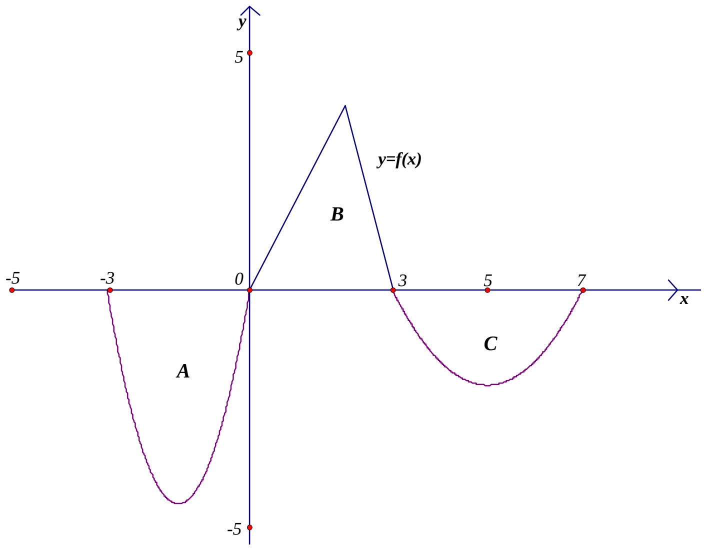


The equation of the semi-circle illustrated alongside is given by

A) $y = \sqrt{9 - x^2}$ B) $y = -\sqrt{9 - x^2}$

C) $y = \sqrt{3 - x^2}$ D) $y = -\sqrt{3 - x^2}$

Question 3



The graph on the right shows the curve $y = f(x)$. The shaded areas are bounded by $y = f(x)$ and the x axis.

Shaded area A is 9 square units, shaded area B is 6 square units and shaded area C is 5 square units.

The value of $\int_{-3}^{7} f(x)dx$ is

A) 8 B) -8

C) 20 D) -20

Question 4

When three marksmen take part in a shooting contest, their chances of hitting the target are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$. Calculate the probability that one, and only one, bullet will hit the target if the three marksmen shoot at the target simultaneously.

A) $\frac{47}{60}$ B) $\frac{1}{3}$ C) $\frac{13}{30}$ D) $\frac{1}{60}$

Question 5

A parabola has the equation $x^2 = 12(8 - y)$. What is the equation of its directrix ?

A) $y = -3$ B) $y = 3$ C) $y = 5$ D) $y = 11$

Question 6

The area between the curve $= \frac{1}{x}$, the x axis and the ordinates $x = 1$ and $x = b$ is equal to 2 square units. The value of b is

A) e B) e^2 C) $2e$ D) 3

Question 7

The roots of the quadratic equation $gx^2 - x + h = 0$ are -1 and 3. The value of h is

A) -6 B) -3 C) $-\frac{3}{2}$ D) 2

Question 8

A table of values made to help sketch the curve of $y = f(x)$ is shown below.

x	0	2	4	6	8
f(x)	7	9	14	4	-3

Given that *f(x)* is continuous over the domain $0 \leq x \leq 8$, the use of Simpson's Rule with five ordinates to estimate $\int_0^8 f(x)dx$ will give the result:

A) 28 B) 40 C) 56 D) 80

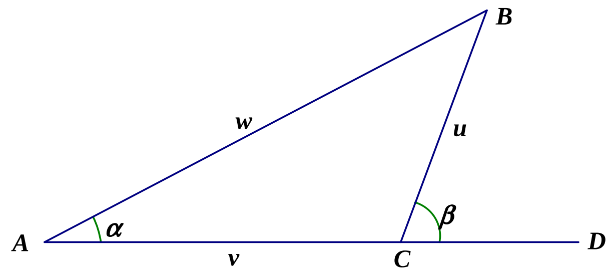
Question 9

A manufacturer increases the price of a car by 15% to a new selling price of \$18860. What was the price of the car before the increase?

A) \$16000 B) \$16031 C) \$16400 D) \$17000

Question 10

If *ACD* is a straight line, $\angle BAC = \alpha$ and $\angle BCD = \beta$ in the diagram shown, which of the following is true ?



A) $w^2 = u^2 + v^2 - 2uv\cos\beta$

B) $w^2 = u^2 + v^2 + 2uv\cos\beta$

C) $u^2 = v^2 + w^2 - 2vw\cos\beta$

D) $\frac{u}{\sin\alpha} = \frac{w}{\cos\beta}$

End of Section I

Section II Total Marks is 90

Attempt Questions 11 – 16
Allow approximately 2 hours & 40 minutes for this section.

Answer all questions, starting each new question on a new sheet of paper with your **Student ID number** in the top right hand corner and the question number on the left hand side of the paper.

All necessary working must be shown in each and every question.

Question 11 (15 Marks) Start a new piece of paper Marks

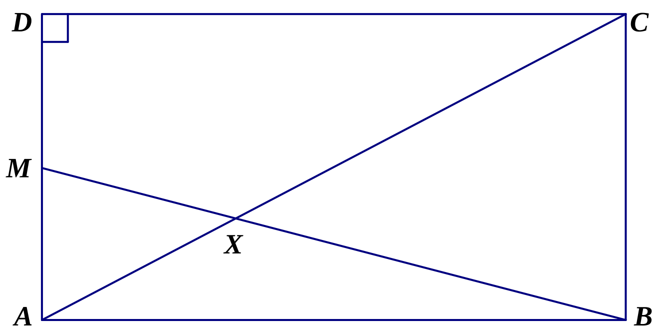
a) Find the value of π^e , correct to three significant figures. **1**

b) The graph of $y = f(x)$ passes through the point $(2,5)$ and $f'(x) = 2x - 3$.
Find $f(x)$. **2**

c) If $\frac{\sqrt{128}-\sqrt{50}}{\sqrt{3}} = \sqrt{k}$, what is the value of k ? (Show working) **2**

d) Find, with a diagram and all necessary working, the equation of the locus of all points which are equidistant from the point $S(2,3)$ and the line $y = 5$. Express your answer in the form $(x - h)^2 = 4A(y - k)$ and hence write down the coordinates of the vertex of the locus. **4**

e) In the diagram, $ABCD$ is a rectangle such that $AB = 2AD$. The point M is the midpoint of AD and the line BM meets AC at X.



i) Copy the diagram and show that the triangles AXM and CXB are similar. **2**

ii) Show that $3CX = 2AC$. **2**

iii) Show that $9(CX)^2 = 5(AB)^2$. **2**

Question 12 (15 Marks) Start a new piece of paper **Marks**

a) Factorise fully $2x^4 + 128x$ **2**

b) Integrate : $\int \frac{(\sqrt{x}+1)^2}{x} dx$ **3**

c) A market gardener plants cabbages in rows. Owing to the wedge shape of his field, the first row has 43 cabbages, the second row has 47 cabbages and each succeeding row has four more cabbages than the previous row.

i) Calculate the number of cabbages in the 12^{th} row. **2**

ii) In this plan, which row would be the first to contain more than 200 cabbages ? **1**

iii) In fact, the farmer finished up only having planted 1065 cabbages. How many rows was that? **2**

d) A team of five students is to be chosen randomly from a class of twelve students. Find the probability that :

i) three particular students *A*, *B* and *C* are all in the team. **2**

ii) students *A* and *B* are chosen but *C* is not. **1**

iii) *A*, *B* and *C* are all omitted from the team. **1**

iv) at least one of the students *A*, *B* or *C* is chosen in the team. **1**

Question 13 (15 Marks) Start a new piece of paper **Marks**

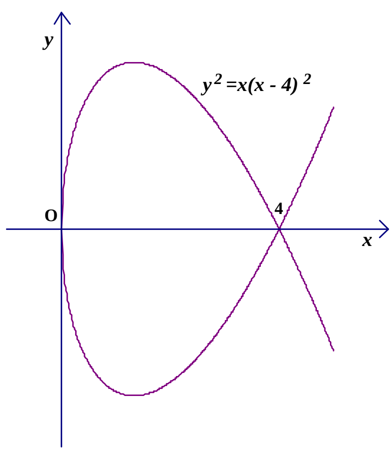
a) Find derivatives (simplifying answers if appropriate) of :

i) πx^{π} **1**

ii) $x\tan 3x$ **2**

iii) $\log_e\left(\frac{x+1}{\sqrt{x-1}}\right)$ **2**

b) The graph on the right shows part of the curve $y^2 = x(x-4)^2$.



i) Find the exact volume of the solid formed when the loop is rotated about the x axis. **2**

ii) Find the area of the loop. **3**

Question 13 is continued on the next page

Question 13 (continued)

c) The rate at which carbon dioxide will be produced by the action of yeast in a dough is given by $\frac{dV}{dt} = \frac{1}{100}(200t - t^2)$ where V cm^3 is the volume of gas produced after t minutes.

i) At what rate is the gas being produced 2 minutes after the yeast begins to work ? **1**

ii) How much carbon dioxide will be produced in the first 5 minutes ? **2**

iii) Assuming that the given formula is only valid while $\frac{dV}{dt}$ is positive, how long will it be before the reaction stops and how much gas will have been produced altogether? **2**

Question 14 (15 Marks) Start a new piece of paper Marks

a) A fish farmer began business on 1st January 2001 with a stock of 100000 fish. He had a contract to supply 14800 fish at a price of \$10 per fish to a retailer at the end of December each year. In the period between January and December each year the number of fish increases by 10%.

i) Find the number of fish remaining after the second harvest in December 2002. **1**

ii) Show that F_n , the number of fish just after the nth harvest, is given by $F_n = 148000 - 48000(1 \cdot 1)^n$ **2**

iii) At the end of which year did the farmer sell his last fish and what was his total income over the life of the business? (NB. In the last season, the farmer will not fully complete his contract but he just sells all that he has.) **3**

b) In this section you will find it useful to draw a set of coordinate axes and update your diagram as information becomes available.

i) Find the equation of the line l which passes through $A(-3,1)$ and $B(0,5)$. **1**

ii) Find the distance from the point $C(2,1)$ to the line l . **1**

iii) Hence, or otherwise, verify that the line l is a tangent to the circle **2**
$$x^2 + y^2 - 4x - 2y - 11 = 0 .$$

iv) Show that the equation of the line through C which is parallel to l is given by $4x - 3y - 5 = 0$ **1**

v) Hence, or otherwise, write down the equation of k, the other tangent to the circle which is parallel to l. **1**

vi) Write down the equations of the two horizontal lines, m and n, which are tangents to this circle. **1**

vii) Find the area of the parallelogram defined by the lines k, l, m and n. **2**

Question 15 (15 Marks) **Start a new piece of paper** **Marks**

a) The mass M grams of a piece of radioactive material at time t years, is decaying according to the equation $\frac{dM}{dt} + kM = 0$ where k is a positive constant.

i) Show that $M = Ae^{-kt}$, where A is a constant, is a solution of this equation. **1**

ii) What is the physical significance of A ? **1**

iii) Given that $A = 50$ and that the mass is 45 grams after 2 years, find the exact value of k. **2**

iv) To the nearest year, what is the half-life of the radioactive material ? (ie. How long does it take for the material to reduce to half of its original mass ?) **2**

b) A particle starts from rest at O and moves along the x axis so that its acceleration after t secs is $(24t - 12t^2)$ m/sec.

i) Find when the particle again returns to O and its velocity at that time. **2**

ii) What is the farthest that the particle travels from O during this interval. **1**

c) A rectangular box, open at the top, is to be constructed out of thin sheet metal on a base which is twice as long as it is wide.

i) The box is to have a volume of 972 cubic units. If its width is x units and its height y units , find a formula for y in terms of x. **1**

ii) Show that the area A square units of sheet metal required is given by

$$A = 2x^2 + \frac{2916}{x}.$$ **2**

iii) Hence find the least area of sheet metal required to make such a box. **3**

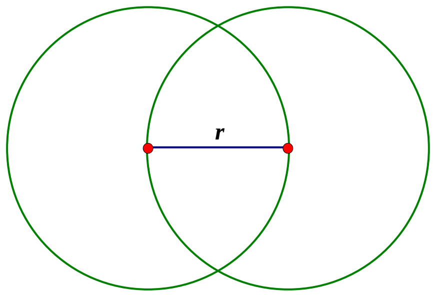
Question 16 will be found on the next page

Question 16 (15 Marks) **Start a new piece of paper** **Marks**

a)

A circle of radius *r* is drawn with its centre on the circumference of another circle of radius *r*. **4**

Find, in terms of *r*, the exact area common to both circles (shaded in the diagram) .



b) i) Draw the graphs of $y = 4\cos x$ and $y = 2 - x$ on the same set of axes for $-2\pi \le x \le 2\pi$. **2**

ii) Explain why all the solutions of the equation $4\cos x = 2 - x$ must lie between $x = -2$ and $x = 6$. **1**

c) Two particles *A* and *B* start moving on the *x* axis at time $t = 0$. The position of particle *A* at time *t* is given by $x_A = -6 + 2t - \frac{1}{2}t^2$ and the position of particle *B* at time *t* is given by $x_B = 4\sin t$.

i) Find expressions for the velocities of the two particles. **2**

ii) Use part (b) of this question to explain why there are exactly two occasions, t_1 and t_2, when the two particles have the same velocity. **1**

iii) Show that the total distance travelled by particle *A* between these two occasions is **3**

$4 - 2(t_1 + t_2) + \frac{1}{2}({t_1}^2 + {t_2}^2)$.

iv) Explain why the two particles can never meet. **2**

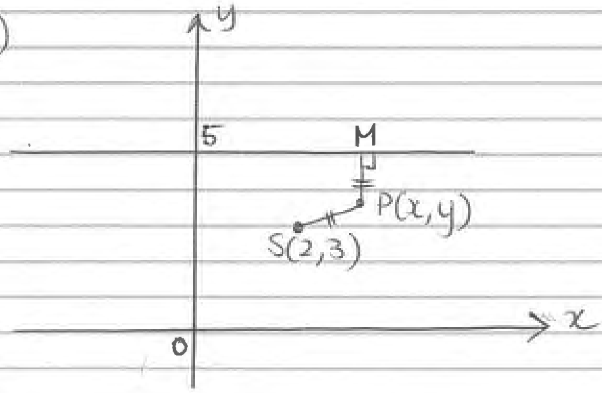
END OF EXAMINATION

MULTIPLE CHOICE

2UNIT TRIAL 2013 (SOLUTIONS & MARKING)

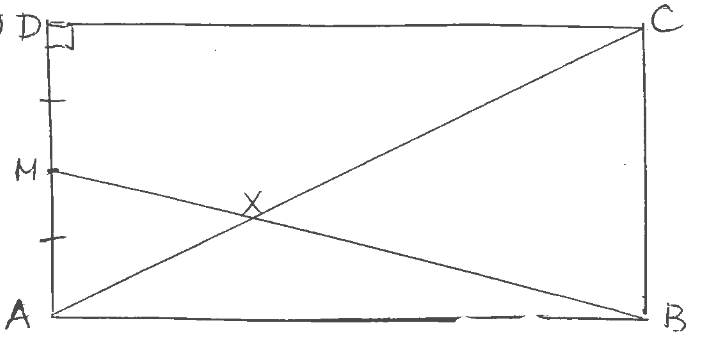
1	2	3	4	5	6	7	8	9	10
C	B	B	C	D	B	C	C	C	B

2013 TRIAL HSC **MATHEMATICS: Question** 11 1 & 3

Suggested Solutions	Marks	Marker's Comments
11a) $\pi^e = 22.459\ldots$		
$= 22.5$ (to 3 sig. figs)	1	correct answer
b) $f(x) = x^2 - 3x + k$	1	integral
$5 = 4 - 6 + k$		
$\therefore k = 7$		
$\therefore f(x) = x^2 - 3x + 7$	1	finding k & stating f(x).
c) $\dfrac{\sqrt{128} - \sqrt{50}}{\sqrt{3}} = \dfrac{8\sqrt{2} - 5\sqrt{2}}{\sqrt{3}}$	1	simplifying surds
$= \dfrac{3\sqrt{2}}{\sqrt{3}}$		
$= \sqrt{3}\sqrt{2}$		
$= \sqrt{6}$		
$\therefore k = 6$	1	finding k
d) 	1	diagram showing point equidistant from y=5 and (2,3).
$PS = PM$		
$\sqrt{(x-2)^2 + (y-3)^2} = 5 - y$		
$(x-2)^2 + y^2 - 6y + 9 = 25 - 10y + y^2$	1	Equating distances.
$(x-2)^2 = 16 - 4y$		
$(x-2)^2 = -4(y-4)$	1	writing equation in form of $(x-h)^2 = 4A(y-k)$
$\therefore$ Vertex is at $(2,4)$		
	1	stating vertex from equation of locus.

Very poorly done! More than half the candidates scored 1 or 0 out of 4 as they failed to derive the locus and find vertex from equation as required.

MATHEMATICS: Question 11

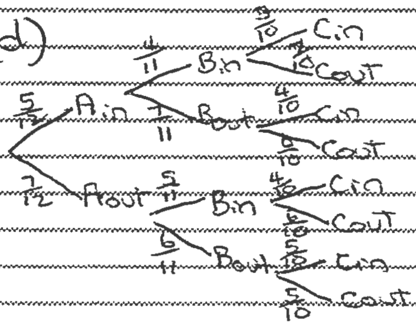
Suggested Solutions	Marks	Marker's Comments
e) 		
(i) $AD \parallel BC$ (opposite sides of rectangle are parallel) In $\triangle AXM$ & $\triangle BXC$ $\angle MAX = \angle BCX$ (alternate angles are equal, $AD \parallel BC$)	1	two reasons
$\angle MXA = \angle CXB$ (vertically opposite angles are equal) $\therefore \triangle AXM \parallel\!\mid \triangle CXB$ (equiangular)	1	two reasons
(ii) $\dfrac{AX}{CX} = \dfrac{AM}{BC}$ (corresponding sides of similar triangles are in same ratio). $CB = DA$ (opposite sides of rectangle) and $AM = \frac{1}{2}AD$ (M is midpoint of AD)	1	reasons
$\therefore \dfrac{AX}{CX} = \dfrac{1}{2}$ $CX = 2AX$ But $AC = AX + XC$ $= 3AX$ $= \dfrac{3CX}{2}$ $\therefore 3CX = 2AC$	1	algebra

MATHEMATICS: Question ...11... 3 of 3

Suggested Solutions	Marks	Marker's Comments
e)(iii) $(AC)^2 = BC^2 + AB^2$ (By Pythagoras' Theorem)	1	
$= \left(\frac{AB}{2}\right)^2 + AB^2$ $\left(\begin{matrix} AD = 2AB \\ \& \ AD = BC \end{matrix}\right)$		
$= \frac{5(AB)^2}{4}$		
$\left(\frac{3CX}{2}\right)^2 = \frac{5(AB)^2}{4}$ (from (ii))		
$\frac{9(CX)^2}{4} = \frac{5(AB)^2}{4}$		
$\therefore\ 9(CX)^2 = 5(AB)^2$	1	

Suggested Solutions	**Marks**	**Marker's Comments**
12 (a) $2x^4+128x = 2x(x^3+64)$ $= 2x(x+4)(x^2-4x+16)$	2	1 mark for $2x(x^3+64)$ 1 mark for final answer
(b) $\int \frac{(\sqrt{x}+1)^2}{x}\,dx = \int \frac{x+2\sqrt{x}+1}{x}\,dx$ $= \int (1+2x^{-\frac{1}{2}}+\frac{1}{x})\,dx$ (1 mark) $= x + 4\sqrt{x} + \ln x + c$ $\frac{1}{2}$ mark each 1 mark	3	1 mark for $\int(1+2x^{-\frac{1}{2}}+\frac{1}{x})\,dx$ and $\frac{1}{2}$ mark for integrating 1 and $2x^{-\frac{1}{2}}$ and 1 mark for integrating $\frac{1}{x}$
(c)(i) 43, 47, 51, ... A.P. as $T_2-T_1=T_3-T_2$ $a=43,\ d=4$ 1 mark for evaluating a & d $\therefore T_{12} = 43+(12-1)\times 4$ $= 43+44$ $= 87$ 1 mark for answer	2	Very few students mentioned that the sequence was an AP. Must mention in future exams before using $T_n = a+(n-1)d$.
(ii) $a+(n-1)d > 200$ $43+(n-1)4 > 200$ $39+4n > 200$ $4n > 161$ $n > 40\frac{1}{4}$ First row to contain more than 200 cabbages is row 41	1	
(iii) $S_n = \frac{n}{2}(2a+(n-1)d)$ $1065 = \frac{n}{2}(86+(n-1)\times 4)$ $2130 = 86n+4n^2-4n$ $2130 = 82n+4n^2$ $\div$ by 2 $2n^2+41n-1065=0$ $n = \frac{-41\pm\sqrt{41^2+4\times 2\times 1065}}{2\times 2}$ $n = \frac{-41\pm\sqrt{10201}}{4}$ $= \frac{-41+101}{4}$ as $n>0$ $n = 15$ $\therefore$ The farmer planted 15 rows	2	1 mark for correct quadratic eqn. 1 mark for answer

2U TRIAL 2013 **MATHEMATICS: Question** 12 2 of 2

Suggested Solutions	Marks	Marker's Comments
12 (cont.) (d) 		OR $n(S) = {}^{12}C_5 = 792$ (i) Sample space received 1 mark for part (i) only
		(i) If ABC are in the team No. of ways $= {}^9C_2 = 36$ $\therefore P(E) = \frac{{}^9C_2}{{}^{12}C_5} = \frac{1}{22}$
(i) $P(A_{in}B_{in}C_{in}) = \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$ 1 mark $= \frac{5}{110}$ $= \frac{1}{22}$ 1 mark	2	(ii) $P(E) = \frac{{}^9C_3}{{}^{12}C_5} = \frac{7}{66}$
(ii) $P(A_{in}B_{in}C_{out}) = \frac{5}{12} \times \frac{4}{11} \times \frac{7}{10}$ $= \frac{7}{66}$ 1 mark	1	$\frac{1}{2}$ mark if they had 2 out of 3 correct numerators
(iii) $P(A_{out}B_{out}C_{out}) = \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10}$ $= \frac{7}{44}$ 1 mark	1	(iii) $P(E) = \frac{{}^9C_5}{{}^{12}C_5} = \frac{7}{44}$
(iv) P(at least 1 student A, B or C) $= 1 - P(ABC\ out)$ $= 1 - \frac{7}{44}$ $= \frac{37}{44}$ 1 mark	1	

2U TRIAL 2013 **MATHEMATICS: Question** 13 (c) 2

Suggested Solutions	Marks	Marker's Comments
13 (a)(i) $\frac{d}{dx}(\pi x^{\pi}) = \pi^2 x^{\pi-1}$	1	
(ii) $\frac{d}{dx}(x\tan 3x) = x \times 3\sec^2 3x + \tan 3x \times 1$ $= 3x\sec^2 3x + \tan 3x$	2	1 mark for $3x\sec^2 3x$ 1 mark for $\tan 3x$ with working
(iii) $\frac{d}{dx}\left[\log_e\left(\frac{x+1}{\sqrt{x-1}}\right)\right] = \frac{d}{dx}\left[\log_e(x+1) - \log_e(x-1)^{\frac{1}{2}}\right]$ $= \frac{1}{x+1} - \frac{1}{2} \times \frac{1}{x-1}$ $= \frac{(2x-2)-(x+1)}{2(x^2-1)}$ $= \frac{x-3}{2(x^2-1)}$ OR $\frac{1}{x+1} - \frac{1}{2(x-1)}$	2	1 mark for applying log law for division 1 mark for either answers
OR $\frac{d}{dx}\left[\log_e\left(\frac{x+1}{\sqrt{x-1}}\right)\right] = \dfrac{\dfrac{\sqrt{x-1} \times 1 - (x+1) \times \frac{1}{2}(x-1)^{-\frac{1}{2}}}{x-1}}{\dfrac{x+1}{\sqrt{x-1}}}$ $= \dfrac{\left(\sqrt{x-1} - \frac{x+1}{2\sqrt{x-1}}\right)}{x-1} \times \frac{\sqrt{x-1}}{x+1}$ $= \dfrac{\dfrac{2(x-1)-(x+1)}{2\sqrt{x-1}}}{x-1} \times \frac{\sqrt{x-1}}{x+1}$ $= \frac{x-3}{2(x^2-1)}$		1 mark for correct $\frac{f'(x)}{f(x)}$ 1 mark for answer
(b) (i) Volume $= \pi\int_0^4 x(x-4)^2\,dx$ $= \pi\int_0^4 (x^3 - 8x^2 + 16x)\,dx$ $= \pi\left[64 - \frac{512}{3} + 128 - 0\right]$ $= \frac{64\pi}{3}\,u^3$ or $21\frac{1}{3}\pi\,u^3$	2	N.B no π − 1 mark 1 mark for the integral in expanded form 1 mark for answer
(ii) Area $= 2\left\|\int_0^4 y\,dx\right\|$ $= 2\left\|\int_0^4 \sqrt{x}(x-4)\,dx\right\|$ $= 2\left\|\int_0^4 (x^{\frac{3}{2}} - 4x^{\frac{1}{2}})\,dx\right\|$ $= 2\left[\frac{2}{5}x^{\frac{5}{2}} - \frac{8}{3}x^{\frac{3}{2}}\right]_0^4$ $= 2\left\|\frac{64}{5} - \frac{64}{3}\right\|$ $\therefore A = \frac{256}{15}$ or $17\frac{1}{15}\,u^2$	3	*NB $\frac{128}{15}u^2$ scored 2 marks 1 mark for integral with ×2 and the absolute value 1 mark for evaluating the integral 1 mark for correct answer

Suggested Solutions	Marks	Marker's Comments
13 (c)(i) $\frac{dV}{dt} = \frac{1}{100}(200t - t^2)$		
When $t = 2$ mins		
$\frac{dV}{dt} = \frac{1}{100}(400-4)$		
$= \frac{396}{100}$		
$\therefore$ Rate is $3.96\text{cm}^3/\text{min}$ OR $\frac{99}{25}\text{cm}^3/\text{min}$	1	
(ii) $\frac{dV}{dt} = \frac{1}{100}(200t - t^2)$		1 mark for integration
$V = \frac{1}{100}\left(100t^2 - \frac{t^3}{3}\right) + C$	2	$\frac{1}{2}$ mark deducted for no "C"
When $V=0$, $t=0$ $\therefore C=0$		OR
$V = \frac{1}{100}\left(100t^2 - \frac{t^3}{3}\right)$		$\frac{1}{100}\int_0^5 (200t - t^2)\,dt$
When $t=5$		$\therefore V = \frac{1}{100}\left[100t^2 - \frac{t^3}{3}\right]_0^5$
$V = \frac{1}{100}\left(2500 - \frac{125}{3}\right)$		$V = \frac{1}{100}\left[2500 - \frac{125}{3}\right]$
$V = 24\frac{7}{12}\text{cm}^3$ or $\frac{295}{12}\text{cm}^3$		$V = \frac{295}{12}$ or $24\frac{7}{12}\text{cm}^3$
		1 mark for answer
(iii) Reaction will stop when $\frac{dV}{dt} = 0$		
$\frac{dV}{dt} = \frac{1}{100}(200t - t^2)$	2	
$0 = \frac{1}{100}(200t - t^2)$		
$\therefore t = 0$ or 200		
When $t = 200$ as $t > 0$		1 mark for $t=200$
$V = \frac{1}{100}\left(100t^2 - \frac{t^3}{3}\right)$		
$= \frac{1}{100}\left(100 \times (200)^2 - \frac{(200)^3}{3}\right)$		
$= \frac{40000}{3}\text{cm}^3$ or $13333.\dot{3}\text{cm}^3$		1 mark for answer
$\therefore$ Reactor stops after 200 minutes in which $\frac{40000}{3}\text{cm}^3$ of gas has be produced		

1/3

MATHEMATICS: Question 14

Suggested Solutions	Marks	Marker's Comments
(i) $F_1 = 100\,000(1.1) - 14800$		
$F_2 = F_1 \times (1.1) - 14800$		
$= [100000(1.1) - 14800]1.1 - 14800$		
$= 100\,000(1.1)^2 - 14800(1.1) - 14800$		right or wrong only.
$= 89\,920$	1	
(ii) $F_3 = F_2 \times 1.1 - 14800$		lost ½ mk if they didn't show and develop F_3.
$= [100000(1.1)^2 - 14800(1.1) - 14800]1.1 - 14800$		
$= 100\,000(1.1)^3 - 14800(1.1)^2 - 14800(1.1) - 14800$		
$= 100\,000(1.1)^3 - 14800(1.1^2 + 1.1 + 1)$		
$F_n = 100\,000(1.1)^n - 14800(1.1^{n-1} + 1.1^{n-2} + \dots + 1)$	½	
$= 100\,000(1.1)^n - 14800 \times \dfrac{1(1.1^n - 1)}{1.1 - 1}$	1	
$= 100\,000(1.1)^n - 148000(1.1)^n + 148\,000$		
$= 148\,000 - 48000(1.1)^n$		
(iii) $F_n = 0$		
$0 = 148000 - 48000(1.1)^n$		
$148000 = 48000(1.1)^n$		
$3.08\dot{3} = 1.1^n$		*A lot of students only half did the question
$\log_{1.1} 3.08\dot{3} = n$	1 mk	
$n = 11.814176$		
$\therefore n = 11$ years		
end of 2012 is the last full season		
$F_{11} = 148000 - 48000(1.1)^{11}$	½ mk	
$= 11050.398$		
$\therefore$ no. of fish at end of 2012 is 11050	½ mk	
so in 2013 there will be 11050×1.1		
$= 12\,155$ fish		
Total Income $= (11\text{yrs} \times 14800 \text{ fish} \times \$10) + (12155 \times \$10)$	1 mk	
$= \$1\,749\,550$		

2/3

MATHEMATICS: Question 14 continued

Suggested Solutions	Marks	Marker's Comments
(b)(i) $m = \frac{5-1}{0-3} = 4/3$	1/2	
$y-5 = 4/3(x-0)$	1/2	
$3y-15 = 4x$		
$\therefore 0 = 4x-3y+15$		
(ii) $a=4$, $b=-3$, $c=15$ $x=2$ $y=1$		
$d = \left\lvert \frac{4\times 2 + -3\times 1 + 15}{\sqrt{16+9}} \right\rvert$		
$= \left\lvert \frac{20}{5} \right\rvert$	1 mk.	
$= 4$ units		
(iii) $x^2+y^2-4x-2y-11=0$		
$x^2-4x+y^2-2y = 11$		
$(x-2)^2+(y-1)^2 = 16$		
circle centre at $(2,1)$ radius $=4$	}	1 mk
since the perpendicular distance is equal to the radius of the circle, the line is a tangent to the circle.	} 1 mk	
(iv) eqn. of line through $C(2,1)$		
$m_2 = 4/3$ (parallel lines have same grad).	1/2	
$y-y_1 = m(x-x_1)$		
$y-1 = 4/3(x-2)$	1/2	
$3y-3 = 4x-8$		
$0 = 4x-3y-5$		
(v) In the form $4x-3y+k=0$ as its parallel to l.		
$4x-3y-25=0$	1	right or wrong
(vi) tangents are $y=5$ and $y=-3$	1	right or wrong

3/3

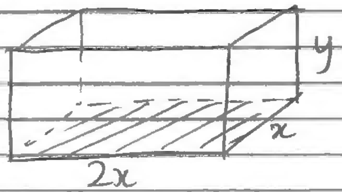
MATHEMATICS: Question 14 cont.		
Suggested Solutions	**Marks**	**Marker's Comments**
(vii) Area $= bh$ height $=$ diameter $= 8$. $= 8 \times 10$ base $= 10$ units $= 80$ units^2 1mk		1mk * really sad that a lot of students did <u>NOT</u> know the area formula.

2013 TRIAL HSC **MATHEMATICS: Question** 15 (i) 3

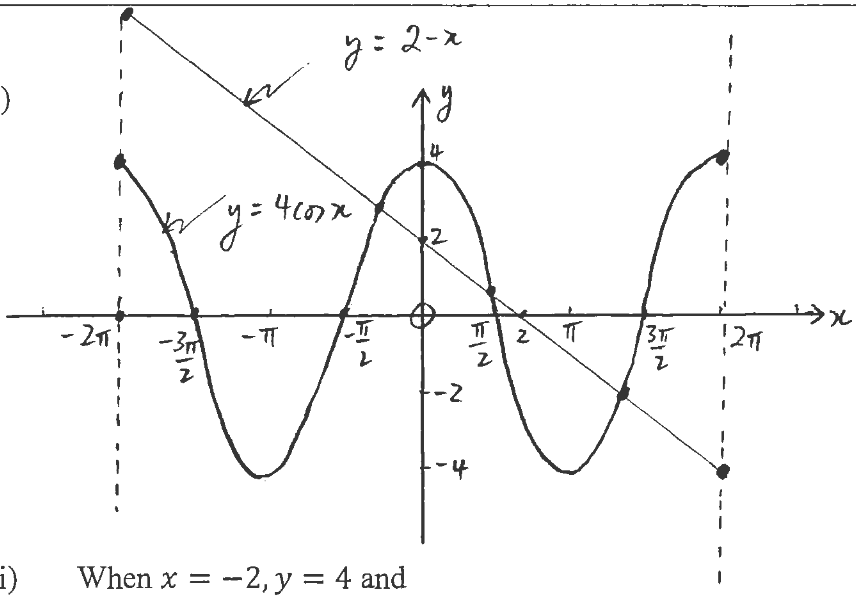
Suggested Solutions	Marks	Marker's Comments
15a) (i) $\frac{dM}{dt} + kM = 0$ $\therefore \frac{dM}{dt} = -kM$ $M = Ae^{-kt}$ $\frac{dM}{dt} = -k \cdot Ae^{-kt}$ $= -kM$ as required $\therefore M = Ae^{-kt}$ is a solution.	1	
(ii) A is the initial mass of material (ie. the mass when $t=0$).	1	
(iii) $M = 50e^{-kt}$ $45 = 50e^{-2k}$ $e^{-2k} = \frac{9}{10}$	1	Simplifying
$-2k = \ln 0.9$ $k = -\frac{1}{2}\ln 0.9$ or $\frac{1}{2}\ln\frac{10}{9}$	1	<u>exact</u> value of k
(iv) When $M = 25$ $25 = 50e^{(\frac{1}{2}\ln 0.9)t}$	1	correct substitution
$\frac{1}{2} = e^{(\frac{1}{2}\ln 0.9)t}$ $(\frac{1}{2}\ln 0.9)t = \ln\frac{1}{2}$ $t = \frac{2\ln\frac{1}{2}}{\ln 0.9}$ $= 13.1576\ldots$		
$\therefore$ Half life is 13 years to nearest year.	1	correct rounding

MATHEMATICS: Question 15 2013		
Suggested Solutions	**Marks**	**Marker's Comments**
b) $\ddot{x} = 24t - 12t^2$ $\dot{x} = 12t^2 - 4t^3 + k$ $\dot{x} = 0$ when $t = 0$ $\therefore k = 0$ $\therefore \dot{x} = 12t^2 - 4t^3$ $x = 4t^3 - t^4 + c$ $x = 0$ when $t = 0$ $\therefore c = 0$ $\therefore x = 4t^3 - t^4$	1	Integrating & finding x.
(i) $x = 0$ $4t^3 - t^4 = 0$ $t^3(4-t) = 0$ $t = 0, 4$ $\therefore$ Particle returns to O at $t = 4s$. Velocity $= 12(4^2) - 4 \times 4^3$ $= -64$	1	$t = 4$ and $v = -64$
$\therefore$ Velocity is -64 m/s (or 64m/s to left) when $t = 4s$.		
(ii) Farthest from O when $v = 0$. $12t^2 - 4t^3 = 0$ $4t^2(3-t) = 0$ $t = 0, 3$ $\therefore$ Particle stops at $t = 3s$. When $t = 3$, $x = 4 \times 27 - 81$ $= 27$		
$\therefore$ Particle's fartherst point from O is 27m.	1	correct answer

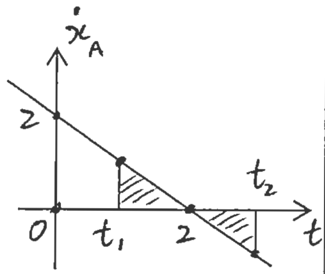
MATHEMATICS: Question 15   39 3

Suggested Solutions	Marks	Marker's Comments
c) 		
(i) Volume $= 2x^2y = 972$ $\therefore y = \dfrac{486}{x^2}$	1	y in terms of x.
(ii) $A = 2xy + 4xy + 2x^2$ $= 2x^2 + 6xy$	1	correct expression for area
$= 2x^2 + 6x \times \dfrac{486}{x^2}$ $= 2x^2 + \dfrac{2916}{x}$	1	substituting y and simplifying.
(iii) $\dfrac{dA}{dx} = 4x - \dfrac{2916}{x^2} = 0$ $x^3 = \dfrac{2916}{4}$ $= 729$ $\therefore x = 9$	1	$\dfrac{dA}{dx} = 0$
$\dfrac{d^2A}{dx^2} = 4 + \dfrac{5832}{x^3} > 0$ when $x = 9$ $\therefore$ Concave up $\therefore$ Minimum area when $x = 9$.	1	finding correct x, substituting in $\dfrac{d^2A}{dx^2}$, for $\dfrac{d^2A}{dx^2} > 0$ & concave up
Minimum area is $2 \times 9^2 + \dfrac{2916}{9} = 486$ sq. units	1	area.

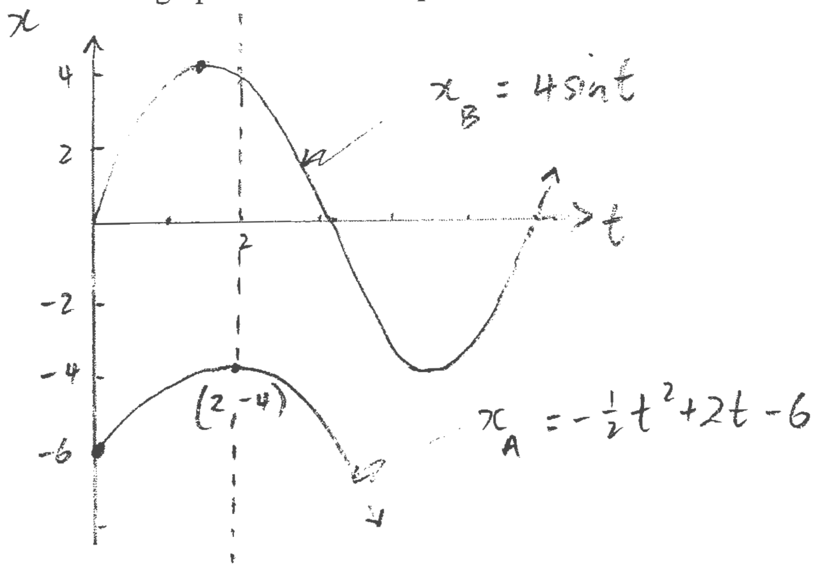
MATHEMATICS QUESTION 16 JRAHS 2U TRIAL 2013

Suggested Solutions	Marks	Marker's Comments
(a) $Area\ of\ shaded\ region =$ $twice\ the\ area\ of\ minor\ segments$	1 1	For a correct sum of areas For correct expression dependent on previous statement
$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$		
But $\theta = 2 \times \frac{\pi}{3}$ .....equilateral triangles	1	For correctly determining the angle at the center (either $\frac{\pi}{3}$ or $\frac{2\pi}{3}$)
Hence $A = \frac{1}{2}r^2(\frac{2\pi}{3} - \sin\frac{2\pi}{3})$		
$= \frac{1}{2}r^2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\ unit^2$	1	For correct final expression in terms of r **NO HALF MARKS**
(b) (i) 	1 1 1	1 mark for both graphs with correct shape For correct intercepts and correct domain for both graphs
(ii) When $x = -2, y = 4$ and when $x = 6, y = -4$ and since the function $4\cos x$ always lie between -4 and 4, the solutions must lie between these points i.e. between -2 and 6		• Poorly answered • Poor use of language • Any sensible explanation

Q16 2013

(c) (i) $v_A = \dot{x}_A = 2 - t$ $v_B = \dot{x}_B = 4\sin t$	1 1	1 mark each, almost all got this out!
(ii) From (b), the equation $2 - x = 4\cos x$ yields 3 solutions i.e. 3 points of intersection. However for $2 - t = 4\cos t$, since $t = time$, and $t > 0$, there are only 2 positive solutions for t.	1	Mention must be made that $t > 0$!
(iii) Method 1: $total\ distance = \int_{t_1}^{2}(2-t)dt + \left\|\int_{2}^{t_2}(2-t)dt\right\|$	1	For correct expression for total distance
$= [2t - \frac{1}{2}t^2]_{t_1}^{2} + \left\|[2t - \frac{1}{2}t^2]_{2}^{t_2}\right\|$	1	Correct integration
$= (4-2) - \left(2t_1 - \frac{1}{2}t_1^2\right) - [\left(2t_2 - \frac{1}{2}t_2^2\right) - (4-2)]$ $= 2 - 2t_1 + \frac{1}{2}t_1^2 - 2t_2 + \frac{1}{2}t_2^2 + 2$ $= 4 - 2(t_1 + t_2) + \frac{1}{2}(t_1^2 + t_2^2)$	1	Correct evaluation of integral
Method 2:  Total distance = signed area under graph from t_1 to 2 and from 2 to t_2 i.e. $distance = \frac{1}{2}(2-t_1)(2-t_1) + \frac{1}{2}(t_2-2)(-(2-t_2)$ $= \frac{1}{2}(4 - 4t_1 + t_1^2) + \frac{1}{2}(t_2^2 - 4t_2 + 4)$ $= 2 - 2t_1 + \frac{1}{2}t_1^2 + \frac{1}{2}t_2^2 - 2t_2 + 2$ $= 4 - 2(t_1 + t_2) + \frac{1}{2}(t_1^2 + t_2^2)$		Generally poorly done! Evidence of "forced" results! Confusion between displacement and distance.
Method 3: The particle stops when $x = 2$. Hence $distance = x_A(2) - x_A(t_1) + x_A(2) - x_A(t_2)$ $= [-6 + 4 - 2] - \left[-6 + 2t_1 - \frac{1}{2}t_1^2\right] + [-6 + 4 - 2] - \left[-6 + 2t_2 - \frac{1}{2}t_2^2\right]$ $= -4 + 6 - 2t_1 + \frac{1}{2}t_1^2 + 6 - 2t_2 + \frac{1}{2}t_2^2 + 4$ $= 4 - 2(t_1 + t_2) + \frac{1}{2}(t_1^2 + t_2^2)$		Many students made the following incorrect formulation: $x_A = x_A(t_2) - x_A(t_1)$, which fails to take into account the two different directions particle A moves. No marks were awarded for this fatal error!

Qn 16 3)3

(iv) $x_A = -6 + 2t - \frac{1}{2}t^2$ $= -\frac{1}{2}[t^2 - 4t + 12)$ $= -\frac{1}{2}(t-2)^2 - 4$ Clearly for $t > 2$ $x_A < -4$ and at $t = 2, x_B > 0$, and hence they will never meet The graph below will help.  	1	